Value at Risk: Assignment 5

Jake Mc Leroth (24913693)

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Contents

Introduction

The following assignment is on risk model risk, based on chapter 6 of Alexander (2009). We look at how model risk arises in our risk models and further make use of a backtesting methodology to evaluate model accuracy. This methodology is paired with various statistical tests to check the validity of the VaR models used.

Question 1

The following question is based on example IV.6.2 in Alexander (2009). In this scenario, we have a position worth R1 000 000 worth of ABSA shares. The goal is to calculate the 1%, 10-day systematic VaR on the 31/12/2024 with the JSE Top 40 as the index. To illustrate how model risk can arise, we use different sample sizes and different methods for estimating β. For simplicity this is all done with normal linear VaR. Recapping how systematic equity VaR is calculated from Alexander (2009):

$$\text{VaR}_{h,\alpha} = \Phi^{-1}(1-\alpha)\hat{\beta}\hat{\sigma}\sqrt{h/250}$$

Where $\hat{\sigma}$ is the yearly volatility. We calculate VaR under the following scenarios:

a) OLS estimation using weekly data since 04/01/2008

b) OLS estimation using weekly data since 08/01/2016

c) OLS estimation using daily data since 04/01/2008

d) OLS estimation using daily data since 08/01/2016

e) EWMA estimation using weekly data with a smoothing constant of 0.95

f) EWMA estimation using weekly data with a smoothing constant of 0.9

g) EWMA estimation using daily data with a smoothing constant of 0.95

h) EWMA estimation using daily data with a smoothing constant of 0.9

The results are summarised in the following table:

Table 1: OLS and EWMA beta, index volatility and VaR for ABSA stock

Metric	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**
β	0.7633	0.8884	0.8353	0.9355	1.1885	1.3155	0.7991	0.6919
Index Volatility (%)	19.67	18.30	20.61	18.77	14.04	13.56	11.12	10.84
VaR (%)	6.99	7.57	8.01	8.17	7.76	8.30	4.13	3.49
VaR (R)	69869.61	75655.48	80116.42	81697.35	77609.83	83005.20	41337.23	34901.47

Looking at the β estimates, the OLS estimates are clearly more similar across the different scenarios compared to the EWMA estimates. The same can be said for the volatility and VaR estimates using OLS. We see that the daily data does result in a higher VaR estimate in this case. Looking at the EWMA estimates, the β values are not very similar, and the volatility estimates are a lot lower than that of the OLS case. In this case, not only does the EWMA methodology seem less robust than OLS, but the VaR estimates are likely underestimated. This table highlights how model risk and estimation risk present themselves in the modelling process. Firstly, the choice of model has a significant effect on our estimates. The different time frames show how scaling the VaR can result in underestimation. Furthermore, the choice of the smoothing parameter is ad hoc, yet the choice has a large effect on the β and VaR estimates.

Question 2

This question is based off of example IV.6.6 in Alexander (2009). Here, we make use of a backtest to evaluate the accuracy of VaR estimates. The idea of a backtest is to estimate VaR through time, and track how often returns exceed the VaR estimates. If we estimate 1% VaR using a specific method, then the method is accurate if returns exceed VaR around 1% of the time. We can also apply statistical tests to further evaluate the accuracy of the VaR model. In this question, we use the independence test as shown in Alexander (2009). The method is shown:

$$LR_{\text{ind}} = \frac{\pi_{\text{obs}}^{n_1} \left(1 - \pi_{\text{obs}}\right)^{n_0}}{\pi_{01}^{n_{01}} \left(1 - \pi_{01}\right)^{n_{00}} \pi_{11}^{n_{11}} \left(1 - \pi_{11}\right)^{n_{10}}}$$

where:

- H_0: exceedences are independent
- n_1 is the number of exceedances
- $n_0 = n - n_1$ (Non-exceedances)
- n_{00}: a non-exceedance followed by another non-exceedance
- n_{01}: a non-exceedance followed by an exceedance
- n_{10}: an exceedance followed by a non-exceedance
- n_{11}: an exceedance followed by another exceedance
- $\pi_{01} = \frac{n_{01}}{n_{00}+n_{01}}$
- $\pi_{11} = \frac{n_{11}}{n_{10}+n_{11}}$

Here, we have the test statistic $-2ln(LR_{ind}) \sim \chi_1^2$. Therefore, if the test statistic is the Chi-Square critical value at a given confidence level, we reject the null hypothesis and conclude that exceedances are not independent.

The context of the question is as follows: we want to calculate the 1% daily VaR per R100 point position on the JSE Top 40. We do this with normal linear VaR. To check our model performance, we run a backtest on 2000 observations from the beginning of 2015 to the end of 2024. From this backtest we perform the independence test to determine whether exceedances are independent or not.

We get the results from the calculations done in the R file **"q2_3_4.R"**. The Chi-Square critical values at 10%, 5% and 1% are 2.705543, 3.841459, and 6.634897 respectively. The test statistic calculated is 4.407794. Hence, we reject the null hypothesis that exceedances are independent at 10% and 5%, but not at 1%. This tells us that there is statistical evidence of exceedances clustering, which is not supposed to happen in an accurate VaR model.

Question 3

This question is based on example IV.6.7 in Alexander (2009). Using the backtest methodology from the previous question, the exceedences can be defined by the following indicator function:

$$I_{\alpha,t+1} = \begin{cases} 1, & \text{if } Y_{t+1} < -\text{VaR}_{1,\alpha,t} \\ 0, & \text{otherwise} \end{cases}$$

This results in a series of ones and zeros to tell us whether VaR was exceeded or not. According to Alexander (2009), a regression model can be used together with the indicator function to tell us if a VaR model is well specified. We fit the following regression model, using the same data as in the previous question as well as the South African Volatility Index (SAVI):

$$I_t = \beta_0 + \beta_1 \Delta\, \text{SAVI}_{t-1} + \varepsilon_t$$

Here, we have the indicator function as the response, and the lagged values of the change in the volatility index as the predictors. We then use and F-test to test the following hypothesis:

$$H_0 : \beta_0 = 0.01, \quad \beta_1 = 0$$

After fitting the model on the data, we get the estimates $\hat{\beta}_0 = 0.018508$ and $\hat{\beta}_1 = 0.005237$. The F-statistic obtained is 5.124272, with a corresponding p-value of 0.006029. Hence, we can reject the null hypothesis at a 99% confidence level. This is evidence of the model not being well specified, as exceedences are not happening 1% of the time.

Question 4

This question is based on example IV.6.8 in Alexander (2009). This question is again based on the same scenario and data as question 2. In this question, we do three coverage tests on VaR estimates that have accounted for volatility clustering.

The methodology of the backtest is the same as before, with the addition of using an exponentially weighted moving average (EWMA) volatility estimate for the VaR calculation. We look at the independence test that was used in question 2, as well as two others. The unconditional coverage test is as follows:

$$LR_{\text{uc}} = \frac{\pi_{\text{exp}}^{n_1} \left(1 - \pi_{\text{exp}}\right)^{n_0}}{\pi_{\text{obs}}^{n_1} \left(1 - \pi_{\text{obs}}\right)^{n_0}}$$

Where $-2ln(LR_{un}) \sim \chi_1^2$. The null hypothesis here is that the number of exceedences is close to α. It does not account for independence of exceedences, however. This can be done in the conditional coverage test, which we can simply calculate from:

$$-2 \ln LR_{\text{cc}} = -2 \ln LR_{\text{uc}} - 2 \ln LR_{\text{ind}}$$

Where $-2ln(LR_{cc}) \sim \chi_2^2$. After calculating the EWMA volatility estimates, the backtest is carried out as before. The test statistic for the unconditional coverage test is 4.378497. For the independence test, we then have 0, since the n_{11} value is 0. Finally, the conditional coverage test statistic is therefore 4.378497. We know from question 2 that the Chi-Square critical values at 10%, 5% and 1% are 2.705543, 3.841459, and 6.634897 respectively. For a Chi-Square distribution of two degrees of freedom, the 10% critical value is 4.60517. Therefore, we reject the null hypothesis of the unconditional coverage test at a confidence level of 95%. For the unconditional coverage test, we fail to even reject at 90%. This is evidence that taking volatility clustering into account has resulted in a more robust VAR model.

Conclusion

This assignment has illustrated important lessons when it comes to the risks we face in how we choose to model VaR. Estimates can be very different across models or even different within the same model but with different data. The backtest methodology with statistical tests has been shown to be an important and useful tool to ensure that VaR models are accurate and robust. These are things that need to be considered when we want to estimate VaR.

References

Alexander, C. (2009). *Market risk analysis, value at risk models.* John Wiley & Sons.